Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 19: Quadratic Functions

In this lesson we continue discovering new functions.

Definition 1. **Quadratic Function:** *A function of the form*

$$f(x) = ax^2 + bx + c,$$

where a, b, and c are real numbers with $a \neq 0$, is called a ***quadratic function****.*

Definition 2. **Standard Form of a Quadratic Function:** *The* ***standard form*** *of a quadratic function is*

$$f(x) = a(x-h)^2 + k.$$

Definition 3. **Converting Quadratic Function into Standard Form:** *Given the quadratic function $f(x) = ax^2 + bx + c$, we rewrite it in the standard form using the following procedure:*

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a(x^2 + \frac{b}{a}x) + c \\ &= a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}) + c \\ &= a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}) - a.\frac{b^2}{4a^2} + c \\ &= a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + c \end{aligned}$$

Comparing the last equation to the previous definition $f(x) = a(x-h)^2 + k$, then $h = -\frac{b}{2a}$ and $k = c - \frac{b^2}{4a}$.

Example 1. *The function $f(x) = -2x^2 + 3x + 1$ is a quadratic function with $a = -2$, $b = 3$ and $c = 1$. Moreover, $h = -\frac{b}{2a} = \frac{3}{4}$ and $k = c - \frac{b^2}{4a} = \frac{17}{8}$ then its* ***standard form*** *is $f(x) = -2(x - \frac{3}{4})^2 + \frac{17}{8}$.*

Definition 4. **Parabola:** *The graph of the function $f(x) = a(x-h)^2 + k$ is called a* ***parabola*** *with* ***vertex*** *(h, k). The vertical line $x = h$ is called the* ***axis*** *of the parabola. If $a > 0$, then the parabola* ***opens up*** *and k is the* ***minimum value*** *of f. If $a < 0$, then the parabola* ***opens down*** *and k is the* ***maximum value*** *of f.*

Example 2. *The function $f(x) = -2(x - \frac{3}{4})^2 + \frac{17}{8}$ is a parabola with vertex $(\frac{3}{4}, \frac{17}{8})$. Since $a = -2 < 0$, then the graph of the function opens down.*

Remark 1. *For the function $f(x) = ax^2 + bx + c$, then $f(h) = f(-\frac{b}{2a}) = c - \frac{b^2}{4a} = k$.*